XE(S-I) M (1)

2016

Time : 3 Hours

Full Marks - 100

Candidates are required to give their answers in their own words as far as practicable

The questions are of equal value

Answer any eight questions, selectig at least one from each group

Group A

1 a) Prove that $A-(B\cup C)=(A-B)\cap(A-C)$

b) Prove that $A \times (B-C)=(A\times B)-(A\times C)$

2. a) Define a partition of a set and prove that a partition of a set defines on equivalence relation in the set.

b) Prove that two equivalence classes are either

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disjoint or equal.

3. a) Define an abelian group and show that the set of all positive rational nos. forms an abelian group and the binary operation $*$ defined by $a * b = \frac{ab}{2}$

b) Show that every cyclic group is an abelian group.

4. a) Define a field with an example.

b) Give an example of a ring without unity.

Group B

5. a) If A, B are two matrices conformable for the multiplication, then prove that $(AB)^{-1}=B^{-1}A^{-1}$

b) Find the rank of the matrix

$$A=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 2 & 1 & 2\end{bmatrix}$$

6. a) Prove that the product of two orthogonal matrices is also orthogogonal.

b) Find the matrices A & B when

$$A+B=\begin{bmatrix}1 & 0 & 2\\ 2 & 2 & 2\\ 1 & 1 & 2\end{bmatrix} \text{ and } A-B=\begin{bmatrix}1 & 4 & 4\\ 4 & 3 & 0\\ -1 & -1 & 2\end{bmatrix}$$

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7. a) Prove that the intersection of two convex sets is also a convex set.

b) Solve the following L.P.P. graphically.

Max.$z = 5x_1+3x_2$

Subject to $3x_1+5x_2\leq 15$

$5x_1+2x_2\leq 10, x_1, x_2 \geq 0$

8. Sove the following L.P.P. using simplex method.

Max $z = 3x_1 + 2x_2$

Subject to $2x_1+x_2 \leq 8$

$x_1 + 3x_2 \leq 15, x_1, x_2 \geq 0$

Group C

9 a) Prove that $\cos x = 1-\frac{x^2}{\lfloor 2}+\frac{x^4}{\lfloor 4}-$ ....%

b) If n is a positive integer, prove that

$(i+i)^n + (1-i)^n = 2^{\frac{n}{2}+1}\text{Cos}\frac{n\Pi}{4}$

10. a) Prove that loge $(a+ib) = \frac{1}{2}\log(a^2+b^2) + i\tan^{-1}(b/a)$

b) Prove that

$\frac{\Pi}{4} = \left(\frac{2}{3}+\frac{1}{7}\right)-\frac{1}{3}\left(\frac{2}{3^3}+\frac{1}{7^3}\right)+\frac{1}{5}\left(\frac{2}{3^5}+\frac{1}{7^5}\right)-$ ....

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11. a) Prove that every convergent sequence is bounded.

b) Show that the sequence

$\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}},$ ....Converges to 2.

12. a) State and prove Cauchy's nth root test.

b) Determine the convergency of the series whose nth term is $\frac{\lfloor n}{n^n}$.

13. a) Prove that ıt a function is differentiable at a point, then it must be continuous at that point.

b) If $f(x) = x^2\sin\left(\frac{1}{x}\right)$, for $x\neq 0$

$= 0$, for $x = 0$

Show that f(x) is continuous and differentiable at $x = 0$ but its derivative is not continuous at $x = 0$.

14. a) Define co-axial circle and obtain the equation of a system of co-axial circle in simplest form.

b) Show that the circles $x^2+y^2-2ax + c = 0$ & $x^2+y^2+2by+c=0$ cut orthogonally.

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15. a) Find the equation of normal at (a cosΦ, b sinΦ) to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

b) If the normal at $(a t_1^2, 2at_1)$ meet the parabolas $y^2 = 4ax$ again in $(at_2, 2at_2)$ then prove that $t_2 = -t_1 - \frac{2}{t_1}$

Group E

16. a) Find the equation of plane in normal form.

b) Find the angle between the lines whose d.c's are given by the equations $l^2+m^2-n^2=0$ and $l+m+n=0$.

17. a) Find the condition that the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{y-\gamma}{n}$ may lie in the plane $ax + by + cz + d=0$

b) Prove that the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are co-planer. Also find the equation of the plane in which they lie.