

2018

Full Marks : 100

Time : 3 hours

The questions are of equal value

Answer **eight** questions, selecting at least **two** from each Group

Group—A

$$\frac{dy}{dx} \cdot \frac{dx}{x}$$

1. (a) If

$$y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$$

then prove that

$$(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} - (n^2 - m^2)y_n = 0$$

(b) If

$$u = \tan^{-1} \frac{x^2 + y^2}{x - y}, \quad x \neq y$$

then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

(2)

2. (a) Evaluate :

$$\lim_{x \rightarrow 0} \left( \frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$$

(b) Find the equation of the tangent at  $(x, y)$  to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

3. (a) Find the Cartesian formula for radius of curvature.

(b) Find those asymptotes of the following curve which are parallel to the coordinate axes  $x^2 y^2 = a^2(x^2 + y^2)$ .

4. (a) Prove that

$$\lim_{n \rightarrow \infty} \left[ \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{3n} \right] = \log 3$$

(b) Integrate

$$\int \sin^5 x \, dx$$

5. (a) Prove that

$$\int_0^{\pi/2} \sin^4 x \cos^2 x \, dx = \frac{\pi}{32}$$

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(Continued)

(3)

(b) Evaluate

$$\int_a^b (x-a)^m (b-x)^n \, dx$$

$m$  and  $n$  being positive integer.

6. Find the surface of the solid generated by the reevaluation of the curve  $x = a \cos^3 t$  and  $y = a \sin^3 t$  about the  $x$ -axis.

7. (a) Solve :

$$\sec^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$$

(b) Solve :

$$x(x-y) \, dy + y^2 \, dx = 0$$

8. (a) Solve :

$$(1+y^2) \, dx = (\tan^{-1} y - x) \, dy$$

(b) Solve :

$$y = px + ap(1-p)$$

9. (a) Find the orthogonal trajectory of  $r = a(1 + \cos \theta)$ .

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(Turn Over)

(4)

(b) Solve the equation :

$$(D^3 - D^2 - 4D + 4)y = e^{3x}$$

Group—B

10. (a) Prove that

$$\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$$

(b) If

$$\bar{r} = \bar{a}e^{nt} + \bar{b}e^{-nt}$$

where  $\bar{a}$  and  $\bar{b}$  are constant vectors, then show that

$$\frac{d^2\bar{r}}{dt^2} - n^2\bar{r} = 0$$

11. (a) If  $\frac{d\bar{a}}{dt} = \bar{r} \times \bar{a}$  and  $\frac{d\bar{b}}{dt} = \bar{r} \times \bar{b}$ , then show that

$$\frac{d}{dt}(\bar{a} \times \bar{b}) = \bar{r} \times (\bar{a} \times \bar{b})$$

(b) If  $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ , then prove that  $\nabla r^n = nr^{n-2}\bar{r}$ .

(5)

12. (a) Prove that

$$\text{div}(\bar{a} \times \bar{b}) = \bar{b} \cdot \text{curl} \bar{a} - \bar{a} \cdot \text{curl} \bar{b}$$

(b) Prove that

$$\text{curl grad } \phi = 0$$

Group—C

13. State and prove the principle of virtual work for a system of coplanar forces acting at different points of a rigid body.

14. Show that a system of coplanar forces acting in one plane at different points of a rigid body can be reduced to a single force through only given point and a single couple.

15. (a) Show that if a particle is acted on by a conservative system of force and be in motion, then the sum of the kinetic and potential energies of the particle remains constant.

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(Turn Over)

(6)

(b) A particle move in a straight line with a constant acceleration  $f$ , starting from same point in the line with a velocity  $u$ , to discuss the motion.

16. (a) A point in a straight line with SHM has velocities  $v_1$  and  $v_2$  when its distances from the centre are  $x_1$  and  $x_2$ . Show that the period of motion is

$$2\pi \sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

(b) A particle is suspended from a fixed point by an elastic string; to discuss the motion.

17. (a) The velocities of a particle along and perpendicular to the radius from a fixed origin are  $\lambda r$  and  $\mu\theta$ , find the path and show that the acceleration along and perpendicular to the radius vector, are

$$\lambda^2 r - \frac{\mu^2 \theta^2}{r} \text{ and } \mu\theta \left( \lambda + \frac{\mu}{r} \right)$$

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(7)

(b) A particle moves in a plane curve, so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve.

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XEV (S-2) - Math