2017

Time : 3 Hours

Full Marks : 100

The questions are of equal value.
Answer ***eight*** *questions, selecting*
at least two from each Group.

Group-A

1. (a) State and prove Maclaurin's Theorem.

   (b) If $y = (\sin^{-1}x)^2$ then prove that

   $$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2y_n = 0$$

2. (a) State and prove Euler's theorem on homogeneous function of three variables.

   (b) Evaluate $\lim\limits_{x \to 0} \dfrac{(1+x)^{\frac{1}{x}} - e}{x}$

3. (a) Write down the equation of the tangent to the curve $y(x^2 + a^2) = ax^2$ at the point $y = \dfrac{a}{4}$.

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   (b) Show that the radius of curvature of a circle is constant.

4. (a) Evaluate from the first principle $\int_a^b x^2 dx$.

   (b) Find the reduction formula for

   $$\int_0^{\pi/2} \cos^m x \cos nx \, dx.$$

5. (a) Find the length of arc of a semi-cubical parabola $ay^2 = x^3$ from the origin to the point (a, a),

   (b) Find the whole area of the curve $r = 2a\cos\theta$.

6. (a) Prove that $\beta(m, n) = \dfrac{\Gamma m \, \Gamma n}{\Gamma(m + n)}$.

   (b) Find the moment of inertia of a thin uniform rod of length 2a about a line through its centre $\perp$r to the rod.

7. (a) Solve $(1 - x^2)(1 - y)dx = xy(1 + y)dy$

   (b) Solve $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan\dfrac{y}{x}$

8. (a) Solve $(1 + y^2)\, dx = (\tan^{-1}y - x)dy$

   (b) Show that the system of confocal co-axial parabolas $y^2 = 4a(x + a)$ belongs to the system itself, a being parameter.

9. (a) Solve $y = -px + x^4p^2$

   (b) Solve $(D^2 - 2D + 4)y = e^x\cos x$

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Group-B

10. (a) Show that $[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}] = [\vec{a}, \vec{b}, \vec{c}]^2$.

(b) Prove that the following four points are coplaner

$2\vec{i}+3\vec{j}-\vec{k}, \vec{i}-2\vec{j}+3\vec{k}, 3\vec{i}+4\vec{j}-2\vec{k}$ and $\vec{i}-6\vec{j}+6\vec{k}$.

11. (a) The necessary and sufficient condition for a vector function $\vec{v}$ of a scalar variable t to have constant direction is $\vec{v}\times\frac{d\vec{v}}{dt}=0$

(b) If $\vec{a}=\vec{j}\sin\theta+\vec{j}\cos\theta+\vec{k}\theta$, $\vec{b}=\vec{i}\cos\theta-\vec{j}\sin\theta-3\vec{k}$, $\vec{c}=2\vec{i}+3\vec{j}-\vec{k}$. Find $\frac{d}{d\theta}\{\vec{a}\times(\vec{b}\times\vec{c})\}$ at $\theta=0$

12. (a) If $\phi = 2x^3y^2z^4$, then find div-grad ϕ.

(b) Prove that $\text{div}(\text{curl}\,\vec{v}) = 0$.

Group-C

13. (a) A particle rest on a smooth curve under the action of any force. Find the position of equilibrium.

(b) The resultant of two forces P and Q acting at a

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certain angle is X and that of P and R acting at the same angle is also X. The resultant of Q and R acting at the same angle of Y. Prove that

$$P=\sqrt{X^2+QR}=\frac{QR(Q+R)}{Q^2+R^2-Y^2}.$$

14. (a) Enumerate the nature of the force which may be omitted in forming the equation of virtual work.

(b) Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A, show that there is a thrust in BD equal to $\frac{W}{\sqrt{3}}$.

15. (a) Derive a relation between work and energy.

(b) Describe principle of linear momentum and derive an expression for linear momentum.

16. (a) Define SHM. Find its periodic time, amplitude and frequency.

(b) Show that the particle executing SHM requires one-sixth of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude.

17. (a) Find the tangential and normal velocities of a particle moving in a plane.

(b) Prove that the energy of a stretched elastic string is equal to half the product of the tension and the extension.

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