Time: 3 Hours

Full Marks: 100

The questions are of equal value.

Answer eight questions, selecting at least two from each Group.

Group-A

- Y. (a) State and prove Maclaurin's Theorem.
 - (b) If $y = (\sin^{-1}x)^2$ then prove that $(1 x^2)y_{n+2} (2n+1)xy_{n+1} n^2y_n = 0$
- (a) State and prove Euler's theorem on homogeneous function of three variables.
 - (b) Evaluate $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}} e}{x}$
- 3. (a) Write down the equation of the tangent to the curve $y(x^2 + a^2) = ax^2$ at the point $y = \frac{a}{4}$.

- (b) Show that the radius of curvature of a circle is constant.
- (b) Find the reduction formula for $\int_{-\infty}^{\pi/2} \cos^m x \cos nx \, dx.$
- 5. (a) Find the length of arc of a semi-cubical parabola $ay^2 = x^3$ from the origin to the point (a, a),
 - (b) Find the whole area of the curve $r = 2a\cos\theta$.
- 6. (a) Prove that $\beta(m, n) = \frac{\lceil m' \rceil \lceil n' \rceil}{\lceil (m+n) \rceil}$.
 - (b) Find the moment of inertia of a thin uniform rod of length 2a about a line through its centre \(\text{Lr}\) to the rod.
- 7. (a) Solve $(1 x^2)(1 y)dx = xy(1 + y)dy$
 - (b) Solve $\frac{dy}{dx} = \frac{y}{x} + \tan \frac{y}{x}$
- 8. (a) Solve $(1 + y^2) dx = (\tan^{-1}y x)dy$
 - (b) Show that the system of confocal co-axial parabolas $y^2 = 4a(x + a)$ belongs to the system itself, a being parameter.
- 9. (a) Solve $y = -px + x^4p^2$
 - (b) Solve $(D^2 2D + 4)y = e^x Cosx$

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- 10. (21) Show that $[\overrightarrow{a} \times \overrightarrow{b}, \overrightarrow{b} \times \overrightarrow{c}, \overrightarrow{c} \times \overrightarrow{a}] = [\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}]^2$.
 - (b) Prove that the following four points are coplaner $2\overrightarrow{i}+3\overrightarrow{j}-\overrightarrow{k}$, $\overrightarrow{i}-2\overrightarrow{j}+3\overrightarrow{k}$, $3\overrightarrow{i}+4\overrightarrow{j}-2\overrightarrow{k}$ and $\overrightarrow{i} - 6 \overrightarrow{j} + 6 \overrightarrow{k}$.
- 11. (a) The necessary and sufficient condition for a vector function v of a scalar variable t to have constant direction is $\overrightarrow{v} \times \frac{d \overrightarrow{v}}{dt} = 0$
 - (b) If $\vec{a} = \vec{j} \sin \theta + \vec{j} \cos \theta + \vec{k} \theta$, $\vec{b} = \vec{i} \cos \theta$ $-\overrightarrow{j}\sin\theta - 3\overrightarrow{k}$ $\overrightarrow{c} = 2\overrightarrow{i} + 3\overrightarrow{j} - \overrightarrow{k}$. $\frac{d}{d\theta} \left\{ \overrightarrow{a} \times (\overrightarrow{b} \times \overrightarrow{c}) \right\} \text{ at } \theta = 0$
- 12. (a) If $\phi = 2x^3y^2z^4$, then find div-grad ϕ .
 - (b) Prove that div(curl v) = 0.

Group-C

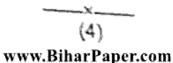
- 13. (a) A particle rest on a smooth curve under the action of any force. Find the position of equilibrium.
- (b) The resultant of two forces P and Q acting at a 1374/79/50/7 (3)(Turn Over)

certain angle is X and that of P and R acting at the same angle is also X. The resultant of Q and R acting at the same angle of Y. Prove that

$$P = \sqrt{X^2 + QR} = \frac{QR(Q + R)}{Q^2 + R^2 - Y^2}.$$

- 14. (a) Enumerate the nature of the force which may be omitted in forming the equation of virtual work.
 - (b) Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A, show that there is a thrust in BD equal to $\frac{W}{\sqrt{3}}$.
- 15, (a) Derive a relation between work and energy.
 - (b) Describe principle of linear momentum and derive an expression for linear momentum.
- 16. (a) Define SHM. Find its periodic time, amplitude and frequency.
 - (b) Show that the particle executing SHM requires one-sixth of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude.
- 17. (a) Find the tangential and normal velocities of a particle moving in a plane.
 - (b) Prove that the energy of a stretched elastic string is equal to half the product of the tension and the extension.

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